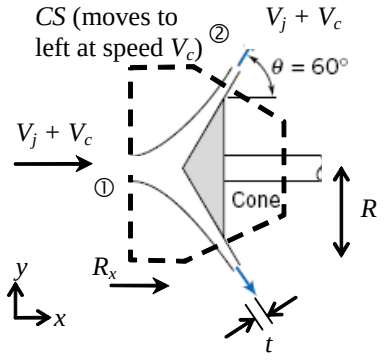
Problem 4.134

[Difficulty: 3]

4.134 Water, in a 4-in. diameter jet with speed of 100 ft/s to the right, is deflected by a cone that moves to the left at 45 ft/s. Determine (a) the thickness of the jet sheet at a radius of 9 in. and (b) the external horizontal force needed to move the cone.



Given: Water jet striking moving cone

Find: Thickness of jet sheet; Force needed to move cone

Solution:

Basic equations: Mass conservation; Momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then $-\rho\cdot V_1\cdot A_1 + \rho\cdot V_2\cdot A_2 = 0$ $\quad -\rho\cdot\left(V_j + V_c\right)\cdot\frac{\pi\cdot D_j^{\,2}}{4} + \rho\cdot\left(V_j + V_c\right)\cdot 2\cdot\pi\cdot R\cdot t = 0$ (Refer to sketch)

Hence $t = \frac{D_j^{\,2}}{8\cdot R}$ $\quad t = \frac{1}{8}\times(4\cdot in)^2\times\frac{1}{9\cdot in}$ $\quad t = 0.222\cdot in$

Using relative velocities, x momentum is

$$R_x = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -\left(V_j + V_c\right)\cdot\left[\rho\cdot\left(V_j + V_c\right)\cdot A_j\right] + \left(V_j + V_c\right)\cdot\cos(\theta)\cdot\left[\rho\cdot\left(V_j + V_c\right)\cdot A_j\right]$$

$$R_x = \rho\left(V_j + V_c\right)^2\cdot A_j\cdot(\cos(\theta) - 1)$$

Using given data

$$R_x = 1.94\cdot\frac{slug}{ft^3}\times\left[(100 + 45)\cdot\frac{ft}{s}\right]^2\times\frac{\pi\cdot\left(\frac{4}{12}\cdot ft\right)^2}{4}\times(\cos(60\cdot deg) - 1)\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad R_x = -1780\cdot lbf$$

Hence the force is 1780 lbf to the left; the upwards equals the weight