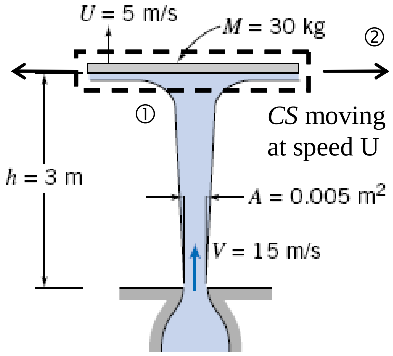
Problem *4.158

[Difficulty: 3]

***4.158** A vertical jet of water impinges on a horizontal disk as shown. The disk assembly mass is 30 kg. When the disk is 3 m above the nozzle exit, it is moving upward at $U = 5$ m/s. Compute the vertical acceleration of the disk at this instant.



Given: Water jet striking moving disk

Find: Acceleration of disk when at a height of 3 m

Solution:

Basic equations: Bernoulli; Momentum flux in z direction (treated as upwards) for linear accelerating CV

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_{S_z} + F_{B_z} - \int_{CV} a_{rf_z}\,\rho\,dV\!\!\!/ = \frac{\partial}{\partial t}\int_{CV} w_{xyz}\,\rho\,dV\!\!\!/ + \int_{CS} w_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow (All in jet)

The Bernoulli equation becomes $\frac{V_0^2}{2} + g\cdot 0 = \frac{V_1^2}{2} + g\cdot(z - z_0)$ $\qquad V_1 = \sqrt{V_0^2 + 2\cdot g\cdot(z_0 - z)}$

$$V_1 = \sqrt{\left(15\cdot\frac{m}{s}\right)^2 + 2\times 9.81\cdot\frac{m}{s^2}\cdot(0-3)\cdot m} \qquad V_1 = 12.9\,\frac{m}{s}$$

The momentum equation becomes

$$-W - M\cdot a_{rfz} = w_1\cdot(-\rho\cdot V_1\cdot A_1) + w_2\cdot(\rho\cdot V_2\cdot A_2) = (V_1 - U)\cdot\left[-\rho\cdot(V_1 - U)\cdot A_1\right] + 0$$

Hence $$a_{rfz} = \frac{\rho\cdot(V_1-U)^2\cdot A_1 - W}{M} = \frac{\rho\cdot(V_1-U)^2\cdot A_1}{M} - g = \frac{\rho\cdot(V_1-U)^2\cdot A_0\cdot\frac{V_0}{V_1}}{M} - g$$ using $V_1\cdot A_1 = V_0\cdot A_0$

$$a_{rfz} = 1000\cdot\frac{kg}{m^3}\times\left[(12.9-5)\cdot\frac{m}{s}\right]^2\times 0.005\cdot m^2\times\frac{15}{12.9}\times\frac{1}{30\cdot kg} - 9.81\cdot\frac{m}{s^2} \qquad a_{rfz} = 2.28\,\frac{m}{s^2}$$