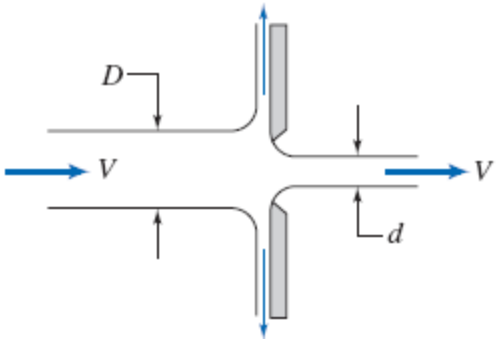
Problem 4.69

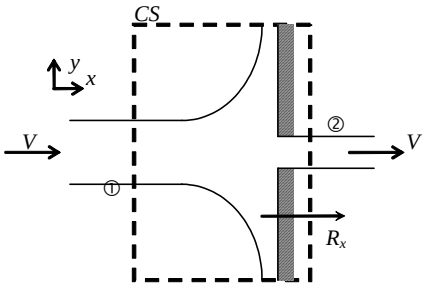
[Difficulty: 2]

4.69 A vertical plate has a sharp-edged orifice at its center. A water jet of speed V strikes the plate concentrically. Obtain an expression for the external force needed to hold the plate in place, if the jet leaving the orifice also has speed V. Evaluate the force for $V = 15$ ft/s, $D = 4$ in., and $d = 1$ in. Plot the required force as a function of diameter ratio for a suitable range of diameter d.



Given: Water jet hitting plate with opening

Find: Force generated on plate; plot force versus diameter d



Solution:

Basic equation: Momentum flux in x direction

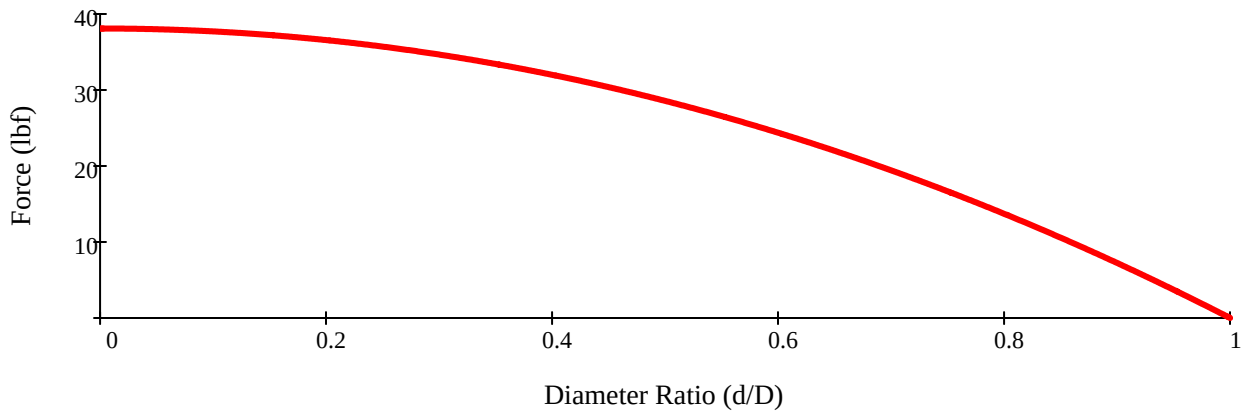
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \sum_{CS} u\,\rho\vec{V}\cdot\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $R_x = u_1\cdot\rho\cdot\left(-u_1\cdot A_1\right) + u_2\cdot\rho\cdot\left(u_2\cdot A_2\right) = -\rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4} + \rho\cdot V^2\cdot\frac{\pi\cdot d^2}{4}$ $\quad R_x = -\frac{\pi\cdot\rho\cdot V^2\cdot D^2}{4}\cdot\left[1-\left(\frac{d}{D}\right)^2\right]$ (1)

For given data $R_x = -\frac{\pi}{4}\cdot 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(15\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\left(\frac{1}{3}\cdot\text{ft}\right)^2\times\left[1-\left(\frac{1}{4}\right)^2\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$ $\quad R_x = -35.7\cdot\text{lbf}$

From Eq 1 (using the absolute value of R_x)



This graph can be plotted in *Excel*