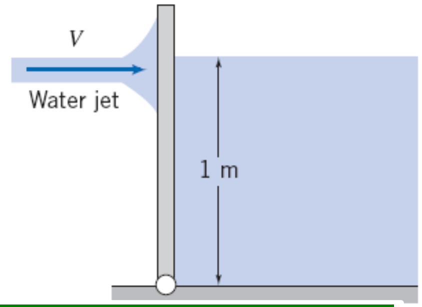
Problem 4.72

[Difficulty: 4]

4.72 A gate is 1 m wide and 1.2 m tall and hinged at the bottom. On one side the gate holds back a 1-m-deep body of water. On the other side, a 5-cm diameter water jet hits the gate at a height of 1 m. What jet speed V is required to hold the gate vertical? What will the required speed be if the body of water is lowered to 0.5 m? What will the required speed be if the water level is lowered to 0.25 m?



Given: Gate held in place by water jet

Find: Required jet speed for various water depths

Solution:

Basic equation: Momentum flux in x direction for the wall

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Note: We use this equation ONLY for the jet impacting the wall. For the hydrostatic force and location we use computing equations

$$F_R = p_c\cdot A \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $$R_x = V\cdot\rho\cdot\left(-V\cdot A_{jet}\right) = -\rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$$

This force is the force generated by the wall on the jet; the force of the jet hitting the wall is then

$$F_{jet} = -R_x = \rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$$ where D is the jet diameter

For the hydrostatic force $$F_R = p_c\cdot A = \rho\cdot g\cdot\frac{h}{2}\cdot h\cdot w = \frac{1}{2}\cdot\rho\cdot g\cdot w\cdot h^2 \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c} = \frac{h}{2} + \frac{\frac{w\cdot h^3}{12}}{w\cdot h\cdot\frac{h}{2}} = \frac{2}{3}\cdot h$$

where h is the water depth and w is the gate width

For the gate, we can take moments about the hinge to obtain $$-F_{jet}\cdot h_{jet} + F_R\cdot(h - y') = -F_{jet}\cdot h_{jet} + F_R\cdot\frac{h}{3} = 0$$

where h_{jet} is the height of the jet from the ground

Hence $$F_{jet} = \rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}\cdot h_{jet} = F_R\cdot\frac{h}{3} = \frac{1}{2}\cdot\rho\cdot g\cdot w\cdot h^2\cdot\frac{h}{3} \qquad V = \sqrt{\frac{2\cdot g\cdot w\cdot h^3}{3\cdot\pi\cdot D^2\cdot h_j}}$$

For the first case (h = 1 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(1\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 28.9\,\frac{m}{s}$$

For the second case (h = 0.5 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(0.5\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 10.2\,\frac{m}{s}$$

For the first case (h = 0.25 m) $$V = \sqrt{\frac{2}{3\cdot\pi}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times(0.25\cdot m)^3\times\left(\frac{1}{0.05\cdot m}\right)^2\times\frac{1}{1\cdot m}} \qquad V = 3.61\,\frac{m}{s}$$