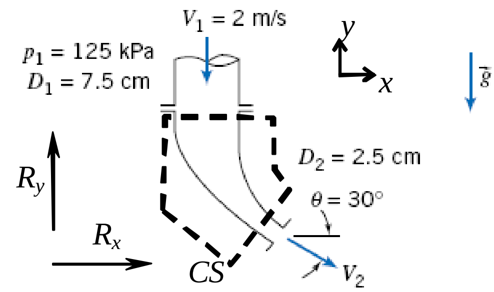
Problem 4.90

[Difficulty: 2]

4.90 A curved nozzle assembly that discharges to the atmosphere is shown. The nozzle mass is 4.5 kg and its internal volume is 0.002 m^3. The fluid is water. Determine the reaction force exerted by the nozzle on the coupling to the inlet pipe.



Given:
Data on nozzle assembly

Find:
Reaction force

Solution:

Basic equation: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,dV + \int_{CS} v\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow CV 3) Uniform flow

For x momentum
$$R_x = V_2\cdot\cos(\theta)\cdot(\rho\cdot V_2\cdot A_2) = \rho\cdot V_2^2\cdot\frac{\pi\cdot D_2^2}{4}\cdot\cos(\theta)$$

From continuity
$$A_1\cdot V_1 = A_2\cdot V_2 \qquad V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2 \qquad V_2 = 2\cdot\frac{m}{s}\times\left(\frac{7.5}{2.5}\right)^2 \qquad V_2 = 18\,\frac{m}{s}$$

Hence
$$R_x = 1000\cdot\frac{kg}{m^3}\times\left(18\cdot\frac{m}{s}\right)^2\times\frac{\pi}{4}\times(0.025\cdot m)^2\times\cos(30\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = 138\cdot N$$

For y momentum
$$R_y - p_1\cdot A_1 - W - \rho\cdot Vol\cdot g = -V_1\cdot(-\rho\cdot V_1\cdot A_1) - V_2\cdot\sin(\theta)\cdot(\rho\cdot V_2\cdot A_2)$$

$$R_y = p_1\cdot\frac{\pi\cdot D_1^2}{4} + W + \rho\cdot Vol\cdot g + \frac{\rho\cdot\pi}{4}\cdot\left(V_1^2\cdot D_1^2 - V_2^2\cdot D_2^2\cdot\sin(\theta)\right)$$

where
$$W = 4.5\cdot kg\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad W = 44.1\,N \qquad Vol = 0.002\cdot m^3$$

Hence
$$R_y = 125\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.075\cdot m)^2}{4} + 44.1\cdot N + 1000\cdot\frac{kg}{m^3}\times 0.002\cdot m^3\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\ \ldots$$
$$+ 1000\cdot\frac{kg}{m^3}\times\frac{\pi}{4}\times\left[\left(2\cdot\frac{m}{s}\right)^2\times(0.075\cdot m)^2 - \left(18\cdot\frac{m}{s}\right)^2\times(0.025\cdot m)^2\times\sin(30\cdot deg)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$R_y = 554\cdot N$$