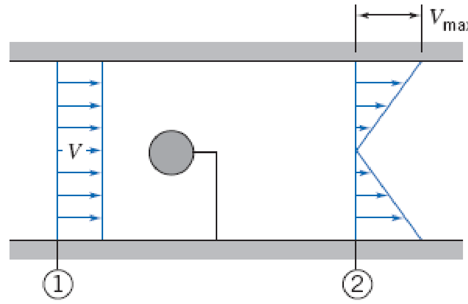
Problem 4.99

[Difficulty: 4]

4.99 A small round object is tested in a 0.75-m diameter wind tunnel. The pressure is uniform across sections ① and ②. The upstream pressure is 30 mm H_2O (gage), the downstream pressure is 15 mm H_2O (gage), and the mean air speed is 12.5 m/s. The velocity profile at section ② is linear; it varies from zero at the tunnel centerline to a maximum at the tunnel wall. Calculate (a) the mass flow rate in the wind tunnel, (b) the maximum velocity at section ②, and (c) the drag of the object and its supporting vane. Neglect viscous resistance at the tunnel wall.



Given: Data on flow in wind tunnel

Find: Mass flow rate in tunnel; Maximum velocity at section 2; Drag on object

Solution: Basic equations: Continuity, and momentum flux in x direction; ideal gas equation

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho \vec{V}\cdot d\vec{A} \qquad p = \rho \cdot R \cdot T$$

Assumptions: 1) Steady flow 2) Uniform density at each section

From continuity $\quad m_{flow} = \rho_1 \cdot V_1 \cdot A_1 = \rho_1 \cdot V_1 \cdot \frac{\pi \cdot D_1^{\,2}}{4}$ $\quad$ where m_{flow} is the mass flow rate

We take ambient conditions for the air density $\quad \rho_{air} = \frac{p_{atm}}{R_{air} \cdot T_{atm}} \qquad \rho_{air} = 101000 \cdot \frac{N}{m^2} \times \frac{kg \cdot K}{286.9 \cdot N \cdot m} \times \frac{1}{293 \cdot K} \qquad \rho_{air} = 1.2 \frac{kg}{m^3}$

$$m_{flow} = 1.2 \cdot \frac{kg}{m^3} \times 12.5 \cdot \frac{m}{s} \times \frac{\pi \cdot (0.75 \cdot m)^2}{4} \qquad m_{flow} = 6.63 \frac{kg}{s}$$

Also $\quad m_{flow} = \int \rho_2 \cdot u_2 \, dA_2 = \rho_{air} \cdot \int_0^R V_{max} \cdot \frac{r}{R} \cdot 2 \cdot \pi \cdot r \, dr = \frac{2 \cdot \pi \cdot \rho_{air} \cdot V_{max}}{R} \cdot \int_0^R r^2 \, dr = \frac{2 \cdot \pi \cdot \rho_{air} \cdot V_{max} \cdot R^2}{3}$

$$V_{max} = \frac{3 \cdot m_{flow}}{2 \cdot \pi \cdot \rho_{air} \cdot R^2} \qquad V_{max} = \frac{3}{2 \cdot \pi} \times 6.63 \cdot \frac{kg}{s} \times \frac{m^3}{1.2 \cdot kg} \times \left(\frac{1}{0.375 \cdot m}\right)^2 \qquad V_{max} = 18.8 \frac{m}{s}$$

For x momentum $\quad R_x + p_1 \cdot A - p_2 \cdot A = V_1 \cdot \left(-\rho_1 \cdot V_1 \cdot A\right) + \int \rho_2 \cdot u_2 \cdot u_2 \, dA_2$

$$R_x = \left(p_2 - p_1\right) \cdot A - V_1 \cdot m_{flow} + \int_0^R \rho_{air} \cdot \left(V_{max} \cdot \frac{r}{R}\right)^2 \cdot 2 \cdot \pi \cdot r \, dr = \left(p_2 - p_1\right) \cdot A - V_1 \cdot m_{flow} + \frac{2 \cdot \pi \cdot \rho_{air} \cdot V_{max}^{\,2}}{R^2} \cdot \int_0^R r^3 \, dr$$

$$R_x = \left(p_2 - p_1\right) \cdot A - V_1 \cdot m_{flow} + \frac{\pi}{2} \cdot \rho_{air} \cdot V_{max}^{\,2} \cdot R^2$$

We also have $\quad p_1 = \rho \cdot g \cdot h_1 \qquad p_1 = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.03 \cdot m \qquad p_1 = 294\,Pa \qquad p_2 = \rho \cdot g \cdot h_2 \qquad p_2 = 147 \cdot Pa$

Hence $\quad R_x = (147 - 294) \cdot \frac{N}{m^2} \times \frac{\pi \cdot (0.75 \cdot m)^2}{4} + \left[-6.63 \cdot \frac{kg}{s} \times 12.5 \cdot \frac{m}{s} + \frac{\pi}{2} \times 1.2 \cdot \frac{kg}{m^3} \times \left(18.8 \cdot \frac{m}{s}\right)^2 \times (0.375 \cdot m)^2\right] \times$ ...

$R_x = -54\,N$ $\quad$ The drag on the object is equal and opposite $\quad F_{drag} = -R_x \qquad F_{drag} = 54.1\,N$